Int. J. Solids Structures Vol. 21, No. 4, pp. 423–424, 1985
Printed in the U.S.A.

0020–7683/85 $3.00 + .00
Pergamon Press Ltd.

BUCKLING AND POSTBUCKLING OF THE LYING SHEET†

R. E. HOBBS‡
Civil Engineering Department, Imperial College of Science and Technology, London SW7 2AZ, United Kingdom

The author is to be congratulated on an interesting contribution to the literature on unilateral contact buckling. There are parallels between his work and studies over the last 40 years and more[1] on the thermally induced buckling of railroad tracks in a vertical plane. This phenomenon, where track/subgrade friction plays a significant role, was a serious problem following the introduction of continuously welded tracks in Europe before the Second World War. Kerr[2] has published a review article with >50 references in this area, comparing (*inter alia*) the behaviour of short test tracks between heavy end restraints and the very long tracks in service. The writer's attention was drawn to this extensive literature when addressing[3] the cognate problem of the thermally induced buckling of subsea pipelines, although (as in some railroad tracks) lateral buckling on the plane against frictional forces is perhaps more relevant to pipeline behaviour. Comparable studies on the performance of concrete pavements and even floating ice sheets have also been reported.

Taking the author's example on page 357, the writer would like to emphasize the different effects of disturbances and initial imperfections. For $F = 5$ and $B = 1$, there are indeed three equilibrium positions: $b = 0$, 0.1 and 0.795. Of these, the first is stable against infinitesimal disturbances but unstable against finite disturbances, the second is unstable and the third is stable. The author's redefinition of critical load as the load below which the sheet cannot buckle under any disturbance is exactly matched by the definition of the "safe force" or equivalently the "safe temperature rise" in the rail track buckling literature.

Now, at least in rail tracks, a force F does not develop instantaneously: it builds up slowly as the rail temperature rises. However, deflections equivalent to b are present because of subgrade irregularities even when $F = 0$. Hence, there is a great practical interest in the effects of initial imperfections b_0 on the buckling behaviour of rail tracks (putting to one side any discussion of the ready source of disturbances provided by an advancing train). The writer[3] has argued on an experimental basis that the effects of initial imperfections b_0 are, in the notation of the author's Fig. 2, as shown in Fig. 1.

Thus, even for small values of b_0, the need for a finite disturbance to the perfect fundamental equilibrium path $b = 0$ is avoided. Instead, as F increases, the system reaches a point such as T, loses stability and snaps to the large displacement equilibrium position U. On further loading, the path US asymptotic to the equilibrium path of the perfect system is followed. As b_0 is increased, the snap TU occurs at lower and lower values of F, until for large enough b_0 the snap is lost altogether and a single-valued nonlinear load displacement path is followed.

In view of these remarks, can the author envisage the extension of his analysis of the lying sheet to include initial imperfection effects? Such an analysis would cast some light on the track-buckling problem. It might also lead to an analytical imperfection study of the track problem, with its frictional effects and (thankfully) small slopes, which would have a definite practical value in establishing tolerable vertical imperfection levels in rail tracks as laid.

† International Journal of Solids and Structures, Vol. 20, No. 4, pp. 351–358.
‡ Senior Lecturer.

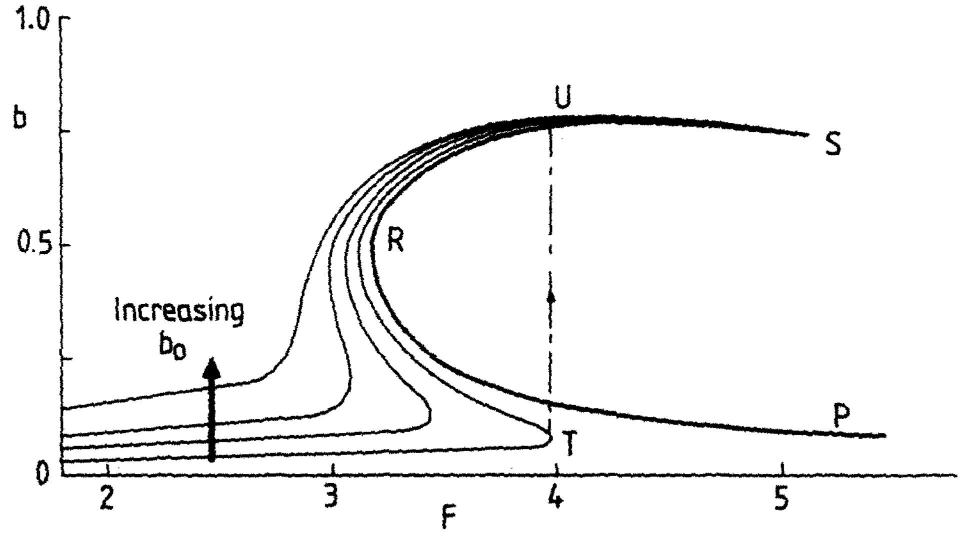

Fig. 1. Values of edge displacement b as a function of horizontal edge force F for $B = 1$ and various initial values of b.

REFERENCES

1. A. Martinet, Flambement des voies sans joints sur ballast et rails de grande longueur (Buckling of tracks without joints on ballast and very long rails: in French). *Rev. Gen. Chemins de Fer* **55**, 212–230 (1936).
2. A. D. Kerr, On the stability of the railroad track in the vertical plane. *Rail Int.* **5**, 132–142 (1974).
3. R. E. Hobbs, In-service buckling of heated pipelines. *Proc. Am. Soc. Civ. Engrs, TE Divn* **110**, 175–189 (1984).